

Topological Mirror Superconductivity

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We demonstrate the existence of topological superconductors (SC) protected by mirror and time reversal (TR) symmetries. D -dimensional ($D=1, 2, 3$) crystalline SCs are characterized by 2^{D-1} independent integer topological invariants, which take the form of mirror Berry phases. These invariants determine the distribution of Majorana modes on a mirror symmetric boundary. The parity of total mirror Berry phase is the \mathbb{Z}_2 index of a class DIII SC, implying that a DIII topological SC with a mirror line must also be a topological mirror SC but not vice versa, and that a DIII SC with a mirror plane is always TR trivial but can be mirror topological. We introduce representative models and suggest experimental signatures in feasible systems. Advances in quantum computing, the case for class D, and topological SCs protected by rotational symmetries are pointed out.

Introduction.— The advent of topological insulators [1, 2] protected by time reversal (TR) symmetry opened the door to the search for other topological states [3–8] with different kinds of symmetries. The idea replacing the winding number of p wave pairing [9–11] by the Berry phase of a single helical band further provided a promising route [12–16] to engineer topological superconductivity using an ordinary superconductor (SC). The hallmark of these class D topological states, governed by TR symmetry breaking and particle hole (PH) redundancy, is the existence of Majorana modes on the boundaries. There is presently a major effort to detect their unique signatures [17–27] such as quantized Andreev reflection and fractional Josephson effect. Recently, realizations of TR invariant topological SCs have also been proposed [27–33] in a variety of settings.

In this letter, we demonstrate the existence of topological SCs that are protected by mirror and TR symmetries and introduce representative models that are experimentally feasible. D -dimensional ($D=1, 2, 3$) crystalline SCs in this symmetry class are characterized by 2^{D-1} integer invariants determined by the mirror Berry phases of the negative energy bands along the mirror and TR invariant lines. These determine the distribution of Majorana modes on a mirror symmetric boundary. Interestingly, the parity of total mirror Berry phase is the \mathbb{Z}_2 index of a class DIII SC, *i.e.*, a fully gapped SC respecting TR symmetry. This relation leads to two important implications: in 1D and 2D a DIII topological SC with a mirror line must also be a topological mirror SC but not vice versa; in 2D and 3D a DIII SC with a mirror plane is always TR trivial but can be mirror topological.

Majorana couplings at the end.— We start by demonstrating the stability of N Majorana Kramers pairs (MKP) at a mirror invariant end. First consider a 1D DIII topological SC, which exhibits a single MKP at the end [27, 28]. In the two dimensional subspace spanned by the two Majorana modes, we may choose a gauge in which the antiunitary TR and PH symmetry operators are $\Theta = \sigma_y K$ and $\Xi = \sigma_x K$, respectively, with K the complex conjugation and σ the Pauli matrices. Now consider N such SCs that physically coincide and each is a

mirror line. This mirror symmetry must be described in this space by $\mathcal{M} = -i\sigma_z$ (up to a sign), since it squares to -1 and commutes with Θ and Ξ . We will demonstrate that there are no mirror symmetric perturbations that can lift the degeneracy of the resulting N MKPs. The Majorana Hamiltonian at the end \mathcal{H}_M must satisfy the constraints of mirror, TR, and PH symmetries: $[\mathcal{H}_M, \mathcal{M}] = \{\mathcal{H}_M, \Pi\} = 0$, where $\Pi \equiv \Xi\Theta = -i\sigma_z$ is the chiral (unitary PH) symmetry operator. Since $\Pi = \mathcal{M}$ it follows that $\mathcal{H}_M = 0$, indicating that *no coupling* among the Majorana modes is allowed without breaking a symmetry. This analysis suggests that, in the presence of mirror-line and TR symmetries, a 1D nodeless SC is characterized by an *integer* invariant that determines N .

Mirror line topological invariant.— In 1D a DIII SC with a mirror line respects *three independent* symmetries:

$$\mathcal{H}_\phi \mathcal{M} = \mathcal{M} \mathcal{H}_\phi, \quad \mathcal{H}_\phi \Theta = \Theta \mathcal{H}_{\bar{\phi}}, \quad \mathcal{H}_\phi \Pi = -\Pi \mathcal{H}_\phi, \quad (1)$$

where $\phi \equiv k$. We are free to choose the pre phase factors of Θ and Ξ such that $\{\Theta, \Pi\} = 0$ and $\Pi^\dagger = \Pi$. The integer invariant is related to the Berry phase of the negative energy states around the 1D Brillouin zone (BZ). Since this Berry phase is gauge dependent, however, the gauge needs to be fixed. This can be accomplished by introducing a continuous deformation that trivializes the Hamiltonian by *relaxing* the chiral symmetry while *keeping* mirror and TR symmetries. We thus add an artificial dimension θ ($-\pi/2 \leq \theta \leq \pi/2$) as follows,

$$\mathcal{H}(\theta, \phi) = \mathcal{H}(\phi) \cos \theta + \Pi \sin \theta. \quad (2)$$

$\mathcal{H}(\theta, \phi)$ inherits the following symmetry constraints

$$\mathcal{M}^{-1} \mathcal{H}(\theta, \phi) \mathcal{M} = \mathcal{H}(\theta, \phi), \quad (3a)$$

$$\Theta^{-1} \mathcal{H}(\theta, \phi) \Theta = \mathcal{H}(-\theta, -\phi), \quad (3b)$$

$$\Pi^{-1} \mathcal{H}(\theta, \phi) \Pi = -\mathcal{H}(-\theta, \phi). \quad (3c)$$

Applying Stokes' theorem, the loop integral of Berry connection [34] along the equator ($\theta = 0$) may be written as the surface integral of Berry curvature [34] $\Omega_{\theta\phi}$ over the north hemisphere ($0 \leq \theta \leq \pi/2$), as shown in Fig. 1(b). This procedure amounts to choosing a gauge in which the

wavefunctions are able to contract into the nonsingular north pole. The mirror symmetry (3a) allows us to label the bands with mirror eigenvalues, and the total and the mirror Berry phases (in units of 2π) of the valence bands are well defined as

$$\gamma_t = \mathcal{C}_N^{v,+} + \mathcal{C}_N^{v,-}, \quad \gamma_m = \mathcal{C}_N^{v,+} - \mathcal{C}_N^{v,-}. \quad (4)$$

Here $\mathcal{C}_{N(S)}^{s,i}$ is the surface integral of $\Omega_{\theta\phi}^{s,i}$ over the north (south) hemisphere normalized by 2π , $s = c(v)$ denotes the conduction (valence) bands, $i = +(-)$ represents the mirror eigenspace with $i\mathcal{M} = +(-)$, and the sum over unspecified band indices is implicit.

The TR symmetry (3b), the *relaxed* chiral symmetry (3c), and the completeness relation for the energy bands $\sum_{n \in \text{all}} |n\rangle\langle n| = 1$ respectively lead to

$$\Omega_{\theta\phi}^{s,i} = -\Omega_{\bar{\theta}\bar{\phi}}^{s,\bar{i}}, \quad \Omega_{\theta\phi}^{s,i} = -\Omega_{\bar{\theta}\bar{\phi}}^{\bar{s},i}, \quad \Omega_{\theta\phi}^{s,i} = -\Omega_{\bar{\theta}\bar{\phi}}^{\bar{s},\bar{i}}. \quad (5)$$

As a result, $\mathcal{C}_{\alpha}^{s,i} = -\mathcal{C}_{\bar{\alpha}}^{s,\bar{i}} = -\mathcal{C}_{\bar{\alpha}}^{\bar{s},i} = -\mathcal{C}_{\alpha}^{\bar{s},\bar{i}}$. In light of the fact that $\sum_{\alpha=S,N} \mathcal{C}_{\alpha}^{s,i}$ is an integer quantized Chern number, we conclude that

$$\gamma_t = 0, \quad \gamma_m = \mathbb{Z}. \quad (6)$$

A qualitative understanding of (6) is possible. For a mirror line, each mirror subspace respects only chiral symmetry and thus has an integer topological invariant, like a 1D insulator in class AIII [3, 4]. In a gauge where the wavefunctions are contractible to a nonsingular point, the valence-band Berry phase uniquely characterizes the winding number of its associated Hamiltonian. It is TR symmetry that requires the two invariants belonging to different mirror subspaces opposite to each other. Consequently, for the valence bands the *total* Berry phase *vanishes* while the *mirror* Berry phase *survives*.

As a consequence, a 1D DIII SC with a mirror line exhibits $|\gamma_m|$ MKPs at the end. For a 2D DIII SC with a mirror line along \hat{x} , there exist *two independent integer* numbers of helical Majorana edge states at the mirror symmetric edge, *i.e.*, $|\gamma_m(0)|$ at $k_y = 0$ and $|\gamma_m(\pi)|$ at $k_y = \pi$. In the presence of more than one mirror lines, different edges may have different Majorana distributions, since different mirrors lead to different invariants. We note that the parity of total mirror Berry phase $(-1)^{\gamma_m}$ in 1D or $(-1)^{\gamma_m(0) \pm \gamma_m(\pi)}$ in 2D is the \mathbb{Z}_2 invariant in class DIII. When $\gamma_m(0) \pm \gamma_m(\pi)$ is odd, at a mirror *asymmetric* edge there also emerges an odd number of helical Majorana edge states protected by TR symmetry. Since a nontrivial \mathbb{Z}_2 index implies a nonzero γ_m a DIII topological SC with a mirror line must also be a topological mirror SC, but not vice versa.

Mirror invariant plane.— We now consider the case for a 2D DIII SC with a mirror plane in which bands can be labeled with mirror eigenvalues. In each mirror subspace, the completeness relation requires $\Omega_{xy}^{v,i} + \Omega_{xy}^{c,i} = 0$ while the chiral symmetry restricts $\Omega_{xy}^{v,i} = \Omega_{xy}^{c,i}$. Therefore,

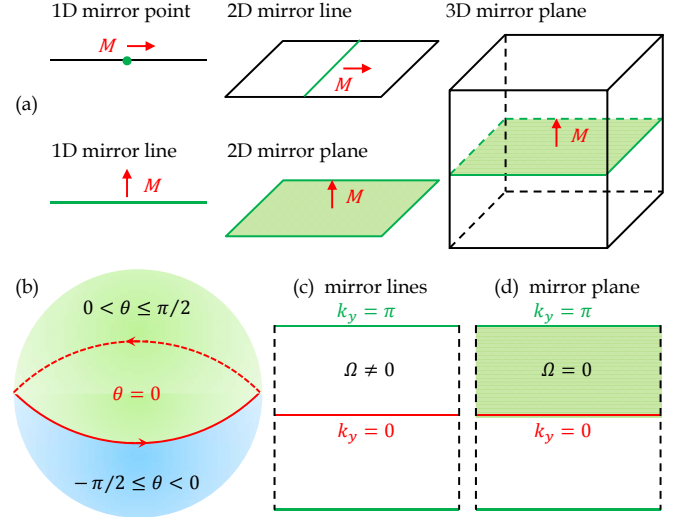


FIG. 1. (a) A sketch of various mirror symmetries. (b) A 1D BZ (red) and an artificial 2D sphere where the chiral symmetry is relaxed. (c) A 2D BZ with mirror invariant lines (red and green) and nontrivial Berry curvature. (d) A 2D BZ as a mirror invariant plane in which Berry curvature vanishes.

$\Omega_{xy}^{s,i} = 0$, implying both the total and the mirror Chern numbers $\mathcal{C}^{v,+} \pm \mathcal{C}^{v,-}$ are zero. Applying Stokes' theorem as shown in Fig. 1(d), the vanishing of the integral of the mirror Berry curvature over a half cylinder with $-\pi \leq k_x \leq \pi$ and $0 \leq k_y \leq \pi$ imposes that

$$\gamma_m(0) = \gamma_m(\pi), \quad (7)$$

i.e., the mirror Berry phases are the same along the two TR invariant lines $k_y = 0$ and π . As a result, the number of helical Majorana edge states at $k_y = 0$ and π is the same. Furthermore, different edges may have different numbers of helical Majorana edge states, indicating that the topological classification of the mirror-plane SCs is $\mathbb{Z} \times \mathbb{Z}$. Again, the parity of total mirror Berry phase $(-1)^{\gamma_m(0) \pm \gamma_m(\pi)}$ is the DIII \mathbb{Z}_2 index. Because of (7), a 2D DIII SC with a mirror plane is always \mathbb{Z}_2 trivial, however, it can be mirror topological.

Interestingly, any change of γ_m from $k_y = 0$ to π would imply the existence of bulk nodes which are topologically protected. On the edge parallel to \hat{y} , there would emerge different numbers of helical Majorana edge states across $k_y = 0$ and π , separated by the projected nodes. This phenomenon is an analog to the zigzag edge state of graphene and the surface Fermi arc of Weyl semimetal.

TABLE I. Topological classification of TR invariant SCs and various mirror SCs in zero to three dimensions.

| | Mirror Point | | Mirror Line | | Mirror Plane | |
|--------|--------------|----------------|----------------|--------------------------------|--------------------------------|----------------|
| Dim | $D = 0$ | $D = 1$ | $D = 1$ | $D = 2$ | $D = 2$ | $D = 3$ |
| DIII | 0 | \mathbb{Z}_2 | \mathbb{Z}_2 | \mathbb{Z}_2 | \mathbb{Z}_2 | \mathbb{Z} |
| Mirror | 0 | 0 | \mathbb{Z} | $\mathbb{Z} \times \mathbb{Z}$ | $\mathbb{Z} \times \mathbb{Z}$ | \mathbb{Z}^4 |

A mirror plane can also exist in a 3D SC. Assuming $\mathcal{M}_z = -i\sigma_z$ mirror symmetry, (7) can be generalized as

$$\gamma_m(k_{||} = 0, k_z) = \gamma_m(k_{||} = \pi, k_z). \quad (8)$$

where $k_z = 0$ or π is a mirror invariant plane and $k_{||}$ denotes k_x or k_y . Clearly, there are *four independent* mirror Berry phases for a 3D mirror SC. At the surface normal to $\hat{k}_{||} \times \hat{k}_z$, there distributes $|\gamma_m(0, 0)|$ helical Majorana surface states centered at $(0, 0)$ and at $(\pi, 0)$, and $|\gamma_m(0, \pi)|$ surface states at $(0, \pi)$ and at (π, π) , respectively. Different mirror symmetries have difference invariants and even one mirror invariant plane may have quite different invariants along different directions, resulting in surface-dependent distributions of Majorana modes. A 3D SC in class DIII is classified by an integer invariant, however, mirror symmetry requires this integer to be zero. In the basis where the chiral symmetry operator is $\Pi = \tau_z$, a 3D DIII SC may be described by $\mathcal{H}_{\mathbf{k}} = Q_{\mathbf{k}}^x \tau_x + Q_{\mathbf{k}}^y \tau_y$, and its integer invariant can be understood as [3, 29]

$$N_w = \frac{1}{24\pi^2} \int d^3\mathbf{k} \epsilon^{ijk} \text{Tr}[Q_{\mathbf{k}}^\dagger \partial_i Q_{\mathbf{k}} Q_{\mathbf{k}}^\dagger \partial_j Q_{\mathbf{k}} Q_{\mathbf{k}}^\dagger \partial_k Q_{\mathbf{k}}], \quad (9)$$

which is the homotopy of $Q_{\mathbf{k}} \equiv Q_{\mathbf{k}}^x - iQ_{\mathbf{k}}^y$. Since $[\mathcal{M}_z, \Pi] = 0$ and $\mathcal{H}_{\tilde{k}_z} = \mathcal{M}_z^{-1} \mathcal{H}_{\mathbf{k}} \mathcal{M}_z$ the integrand of (9) transforms as a pseudoscalar under mirror operation, *i.e.*, $\rho_w(\mathcal{M}_z \mathbf{k}) = -\rho_w(\mathbf{k})$. This leads to $N_w = 0$, as in the inversion symmetric case. Therefore, a 3D DIII SC with a mirror plane must be TR trivial, however, it can be mirror topological.

A 0D or 1D DIII SC with a mirror point can be also considered. At the mirror invariant point, the number of positive and negative energy states is the same because of PH symmetry, and only the \mathbb{Z}_2 parity of the number of negative energy states can be well defined, given that the total charge is not conserved. TR symmetry further requires an even parity because of Kramers degeneracy. Therefore, there is no topological classification for a mirror invariant point. Table I summarizes our results in different mirror classes and in different dimensions.

Representative models.— The simplest experimentally feasible example [27] of topological mirror superconductivity is a Rashba wire proximity-coupled to a *nodeless* s_{\pm} wave SC, *e.g.*, an iron-based SC [35, 36]. Such a hybrid system can be described by

$$\begin{aligned} \mathcal{H} = & (-2t \cos k_x + 2\lambda_R \sin k_x \sigma_z - \mu) \tau_z \\ & + (\Delta_0 + 2\Delta_1 \cos k_x) \tau_x, \end{aligned} \quad (10)$$

where t is the nearest neighbor hopping, μ is the chemical potential, and λ_R is the strength of Rashba spin-orbit coupling. σ are the Pauli matrices of electron spin while τ are the Pauli matrices in Nambu PH notation. The order parameters Δ_0 and Δ_1 , induced by the proximity effect, lead to a s_{\pm} wave pairing potential that switches sign between $k_x = 0$ and π when they satisfy $|\Delta_0| < 2|\Delta_1|$. For convenience we will assume $\Delta_0 = 0$ and $\Delta_1 > 0$. (10) has TR ($\Theta = \sigma_y K$), PH ($\Xi = i\sigma_y \tau_y K$), and

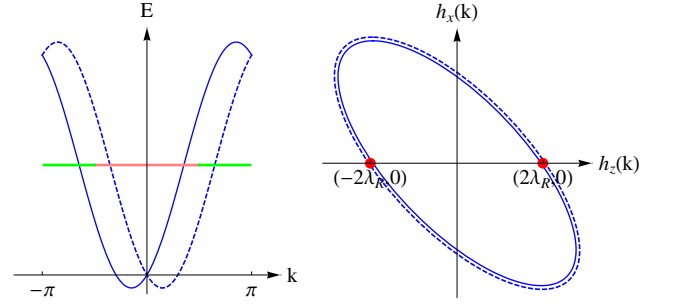


FIG. 2. Left: the two energy bands split by Rashba spin-orbit coupling and labeled by $i\mathcal{M}_z = \pm 1$; the pink and green lines depict that the pairing potential switch signs at $k = \pm\pi/2$. Right: the winding of the vector $\mathbf{h}^{\pm}(\mu = 0)$ as k varies from 0 to 2π , clockwise (counterclockwise) for $\mathbf{h}^{+(-)}$. Solid and dashed lines distinguish the cases for $i\mathcal{M}_z = \pm 1$.

chiral ($\Pi = \tau_y$) symmetries. In addition, there is a mirror ($\mathcal{M}_z = -i\sigma_z$) symmetry in the bulk and at the ends. When $|\mu| < 2\lambda_R$ a positive (negative) pairing is induced for the inner (outer) pair of Fermi points, realizing [27] a 1D \mathbb{Z}_2 topological SC in class DIII. The \mathbb{Z}_2 invariant can alternatively be understood using mirror Berry phase. Due to the mirror symmetry, (10) decomposes into $\mathbf{h}^{\pm} \cdot \boldsymbol{\tau}$ with \pm denoting $i\mathcal{M}_z = \pm 1$. As shown in Fig. 2, (h_z^{\pm}, h_x^{\pm}) have fixed points $(\pm 2\lambda_R - \mu, 0)$ at $k_x = \pm\pi/2$. For the case when $|\mu| < 2\lambda_R$, the windings of \mathbf{h}^{\pm} both enclose the origin *once* but with *opposite* orientations as k varies from 0 to 2π . Therefore, the mirror Berry phase is one and there is one MKP at the end, as predicted earlier.

The higher degeneracies of Majorana modes promoted by mirror symmetry are more amazing. A Majorana quartet can be achieved [27] on the π -junction of two 1D DIII topological SCs with a common mirror symmetry. Indeed a few-wire system with each described by (10) realizes a $|\gamma_m| > 1$ case, as long as their couplings do not close the bulk gap or break mirror symmetry. This few-wire system is equivalent to generalizing (10) to $\mathcal{H}_N = \lambda_N \sin(Nk) \sigma_z \tau_z + \Delta_N \cos(Nk) \tau_x$, where we set $\mu = t = 0$. Conceivably, the first (second) term arises from the N th-neighbor spin-orbit coupling (pairing) in an effective 1D chain. \mathcal{H}_N provides a representative 1D topological mirror SC with $\gamma_m = N$.

Furthermore, \mathcal{H}_N can be readily generalized to

$$\begin{aligned} \mathcal{H} = & [\lambda_N \sin(Nk_x) \sigma_z - \lambda_M \sin(Mk_z) \sigma_x] \tau_z \\ & + [\Delta_0 + \Delta_N \cos(Nk_x) + \Delta_M \cos(Mk_z)] \tau_x, \end{aligned} \quad (11)$$

which describes a 2D DIII SC with a \mathcal{M}_z mirror line. When $\Delta_0 \neq \pm\Delta_N \pm \Delta_M$, (11) has a full gap. Along the mirror invariant lines $k_z = 0$ and π , we obtain

$$\gamma_m = N \text{sgn}(\lambda_N \Delta_N) \Theta(|\Delta_N| - |\Delta_0 + e^{iMk_z} \Delta_M|). \quad (12)$$

Note that (11) also has \mathcal{M}_x mirror symmetry [37]. The $N = M = 1$ case describes a Rashba layer proximity-coupled to a *nodeless* s_{\pm} wave SC, in which one mirror Berry phase vanishes while the other is one when

$|\Delta_0| < 2|\Delta_1|$. Because the total mirror Berry phase has an odd parity, this 2D topological mirror SC is also a DIII topological SC, consistent with a previous [27] result.

Another experimentally feasible model that describes a 2D DIII SC with mirror lines [38, 39] is

$$\mathcal{H} = [\beta k_{||}^2 - \mu + \alpha(\mathbf{k} \times \boldsymbol{\sigma})_z s_z + m s_x] \tau_z + \Delta s_z \tau_x, \quad (13)$$

which can be realized in two physical systems. The first is a thin film [40] of topological insulator with a mirror symmetry ($\mathcal{M}_{||} = -i\sigma_{||}$), *e.g.*, the Bi_2Se_3 family. The α -term describes the top and bottom ($s_z = \pm 1$) surface states that have opposite helicities, while the *smaller* β -term denotes the surface state curvature which is the same on both surfaces [41]. m is a trivial mass due to the finite size tunneling between the two surfaces. When $|\Delta| > |m|$, the system is a DIII topological SC since the two surfaces have opposite pairing. The second system is a Rashba bilayer [33], *e.g.*, the two interfacial 2DEGs ($s_z = \pm 1$) of $\text{LaAlO}_3\text{-SrTiO}_3\text{-LaAlO}_3$ sandwich. The β -term is the kinetic energy of each 2DEG, while the *smaller* α -term represents the Rashba spin-orbit coupling [42] that switches sign on the two interfaces as they are exposed to opposite local electric fields. m is a small gap at $k = 0$ because of interlayer hybridization. When $|\mu| < |m|$ and $\Delta \neq 0$, only the two outer helical bands are present at Fermi energy and they acquire opposite pairing, realizing a DIII topological SC. The odd-parity pairing in (13) can be engineered via a π -junction [12] or may be favored by repulsive interactions [33]. In either case the \mathbb{Z}_2 topological SC must also be a topological mirror SC as we have proved earlier. Near $k_{||} = 0$ the pairing $\sim s_z \tau_x$ is opposite for different surfaces or layers while at $k_{||} = \infty$ the gap is trivial. Therefore, $\gamma_m(0)$ is one for one of the two orbitals and the other three γ_m 's all vanish.

Finally we consider models for topological mirror-plane SCs. When $\delta \cos k_y \tau_z$ is added, the model (11) has a mirror plane. For a sufficiently small δ , the spectrum is fully gapped and all results about (11) still hold at $k_y = 0$ and π . The model (13) also has a mirror plane when modified to $\mathcal{H} = \alpha[\sin(Nk_x)\sigma_y - \sin(Mk_y)\sigma_x]s_z\tau_z + \sin k_z s_y \tau_z + \Delta \cos Mk_y \tau_x$ with $\mu = 0$. Consider the $k_z = 0$ (or π) line on the $k_x = 0$ (or π) mirror invariant plane, the two mirror Berry phases are $\pm M$, with opposite signs for $s_z = \pm 1$ orbitals, and hence a zero total mirror Berry phase. On the $k_y = 0$ (or π) mirror invariant plane, all mirror Berry phases vanish, since the pairing is uniform. Analysis of a 2D mirror-plane SC is similar, when k_x -terms are cut off in either model. These mirror-plane SCs are always TR trivial but can be mirror topological.

Discussion.— We have only illustrated the physics of topological mirror superconductivity in simple lattices, but our theory readily applies to any crystal structure. A topological mirror SC and its Majorana multiplet are robust if by average [44, 45] the disorder respects mirror and TR symmetries. Besides a mirror fractional Josephson effect [27], the MKPs also lead to a quantized An-

dreev reflection producing a pronounced Zero bias conductance peak in tunneling spectroscopy. Adding a Zeeman field proportional to the mirror operator do not change the mirror Berry phase, as long as the reduced gap remains open. However, the Andreev bound states would be lifted from but still symmetric around zero energy. As a result, such a field would reduce and split the peak. In sharp contrast, other Zeeman fields destroy the peak without splitting it.

0D Majorana modes in a 1D topological SC network give hope for fault-tolerant quantum computing [43]. Manipulating different Majorana modes without coupling or dephasing them is a challenging but necessary task, which may be solved by adding TR and mirror symmetries. The TR symmetry likely provides an Anderson's theorem to mitigate the role of disorder in bulk superconductivity; a mirror symmetry further allows the existence of multiple Majorana modes and more importantly prohibits any coupling among them. When respecting the same mirror symmetry, the number of MKPs at the linked end of two topological SCs, with the same (opposite) sign(s) of their mirror Berry phases, are additive (subtractive) for two left or two right ends and subtractive (additive) for one left and one right ends.

It is also fascinating to consider other crystalline symmetries. While inversion is always broken by a boundary, a rotational symmetry can be respected. It turns out that a rotational symmetry plays at least two different roles. For a given topological mirror SC, different mirror symmetries may be related by a rotational symmetry and their mirror Berry phases are hence the same. On the other hand, a rotational symmetry itself can give rise to topological superconductivity with an *integer* quantized *rotational* Berry phase. For instance, with $\sin k_x \sigma_z$ replaced by $\sin k_x \sigma_x$ and $k_x \sigma_y - k_y \sigma_x$ replaced by $k_x \sigma_x + k_y \sigma_y$, our models (10) and (13) respectively describe a 1D and a 2D topological SC protected by their $C_2(\hat{x})$ symmetries. Finally we note that for a class D SC with a mirror plane where the mirror Chern number exists, the classification is \mathcal{Z} in 2D and $\mathcal{Z} \times \mathcal{Z}$ in 3D. Topological mirror superconductivity will likely open up new horizons for Majorana physics and even more topological crystalline SCs await to be discovered.

Note added.— After the finalization of this work, a complementary and independent study [46] appeared.

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